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Seiberg-Witten theory for the asymptotic free rank three tensors of $SU(N)$

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Abstract

We here give a first indication that there exists a Seiberg-Witten curve for $SU(N)$ Seiberg-Witten theory with matter transforming in the totally anti-symmetric rank three tensor representation. We present a derivation of the leading order hyperelliptic approximation of a curve for this case. Since we are only interested in the asymptotic free theory we are restricted to $N = 6, 7, 8$. The derivation is carried out by reversed engineering starting from the known form of the prepotential at tree level. We also predict the form of the one instanton correction to the prepotential.

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A lot of effort has been spent over the years in Seiberg-Witten theory and all asymptotic free cases has been accounted for except $SU(N)$ with matter transforming in the totally antisymmetric rank three tensor representation. Curves have been derived by generalisations [1] of Seiberg and Wittens original work [2], by first principles from M-theory [3] and most recently by reversed engineering [4]. Furthermore, results that can be subject to tests of those curves have been made available for all cases [5, 6, 4].

Here we start a program to determine the Seiberg-Witten curve for $SU(N)$ with matter transforming in the totally antisymmetric rank three tensor representation. Up to this point no information has been presented for this case and fewer clues are available than for e.g. the case of two antisymmetric two tensors [4] that was derived by similar methods.

One of the essential ingredients that characterise the Seiberg-Witten theory is a particular residue function denoted $S(x)$ below. For the case studied here there is no obvious candidate for this function in contrast to e.g. the rank two tensor representations for $SU(N)$. For the rank two tensor representations there were also involutions symmetries which from a M-theory perspective implemented themselves using known objects such as orientifolds. This involution constrained the form of the curve. For the case at hand the realisation of such a symmetry at M-theory level is not known and hence the form of the curve seems less constrained.

Never the less one may use the known form of the prepotential at tree level and simple arguments like dimension analysis to find a candidate for a curve for the $SU(N)$ Seiberg-Witten theory with matter transforming in the totally antisymmetric rank three tensor representation. This curve should be regarded as the lowest order approximation of a full curve and as a first step toward the true answer.

The weights of the totally antisymmetric three tensor representation of SU(N) may be parameterised by

$$e_i + e_j + e_k \qquad\qquad i < j < k = 1, ..., N. \tag{1}$$

where e_i are the weights of the defining representation. The Dynkin index of the three tensor is given by

$$I_{Dynkin} = \frac{(N-2)(N-3)}{2} \tag{2}$$

and hence asymptotic freedom, which requires $2N - I_{Dynkin} \geq 0$, restrict us to $N = 6, 7$ and 8. We may also add up to 5 and 3 defining representations for $SU(6)$ and $SU(7)$ respectively, while $SU(8)$ does not allow for any more matter.

Take the hyperelliptic curve

$$y^2 + 2A(x)y + L^2 B(x) = 0, \qquad L^2 = \Lambda^{2N-(N-2)(N-3)/2}, \tag{3}$$

where Λ is the dynamical scale of the theory. It is sometimes convenient to use the alternative form

$$y^2 = (A(x))^2 - L^2 B(x) \tag{4}$$

and we will interchangeably use the notation hyper elliptic approximation for either of those two forms.

The purpose of this paper is to find out whether or not there exists functions $A(x)$ and $B(x)$ such that this curve reproduces the tree level prepotential using the Seiberg-Witten method. The tree level part of the prepotential is proportional to

$$\sum_{i,j=1}^{N} (e_i - e_j)^2 ln \left(\frac{(e_i - e_j)^2}{\Lambda^2} \right) - \sum_{i<j<k=1}^{N} (e_i + e_j + e_k)^2 ln \left(\frac{(e_i + e_j + e_k)^2}{\Lambda^2} \right). \tag{5}$$

and starting from a hyperelliptic curve there is a well known prescription [5] how to check this result. Here the objective is to revese this process.

We expect to be able to find appropriate functions for the curve by reverse engineering following the method developed in [4]. Reversed engineering makes use of symmetries, functional forms, and dimensional restrictions to find a candidate for a curve. The restriction available apart from the form of the tree level prepotential (5) is the R-parity of the 1-instanton correction or equivalently the curve. Although the form of the 1-instanton contribution to the prepotential is not known it will follow from the form of $A(x)$ and $B(x)$ and since the R-parity of the 1-instanton term is known this puts restrictions on $A(x)$ and $B(x)$.

When integrating out the matter one should find pure Yang-Mills and hence we expect the usual $\prod_{i=1}^{N}(x - e_i)$ to be a part of $A(x)$. The form of the weights indicates that $B(x)$ should contain a factor of $\prod(x + e_i + e_j)$. Here there is no obvious choice for the range of indices. Options available are $i, j = 1, ..., N$, $i \leq j = 1, ...N$ or $i < j = 1, ...N$ and although there is no obvious candidate the minimal choice seems the most attractive one for computational reasons. Hence we begin with this choice and we will subsequently comment on the other possible choices. By inspection it is clear that any of the choices of $B(x)$ given above will give too many weights and those have to be corrected for in some way.

Before we continue we must carry out the calculations to find what kind of logarithmic terms follow from the choices

$$A(x) = \prod_{i=1}^{N}(x - e_i) \qquad\qquad B(x) = \prod_{i<j=1}^{N}(x + e_i + e_j). \tag{6}$$

where we for a moment forget R-parity restrictions. Following Seiberg-Witten [2] we would like to calculate the periods a_i and dual periods a_i^D since the latter are related to the prepotential F via

$$a_i^D = \frac{\partial F}{\partial a_i}. \tag{7}$$

The periods and dual periods follow from the usual expression in terms of integrals over the cycles A_k and dual cycles B_k as

$$a_i = \oint_{A_k} \lambda dx \qquad\qquad a_i^D = \oint_{B_k} \lambda dx \qquad\qquad \lambda = \frac{dy}{y}. \tag{8}$$

The cycles follow from the shape of the curve and the full set of cycles is not known without knowing the exact form of the curve. However, we can find a subset of cycles and find their corresponding periods' contribution to the tree level prepotential. Additional cycles required to meet the constraints on R-parity and correct number of weights will give us additional contributions.

In order to carry out the integrals we need the branchcut structure of the curve. In its current form the curve have N branchcuts and the branchpoints follow from the constraint that those are the common points of the two sheets y_+ and y_- where

$$y_\pm = \sqrt{A^2(x) - L^2 B(x)}. \tag{9}$$

For convenience we introduce the residue function

$$S(x) = \frac{B(x)}{A^2(x)} \qquad\qquad S_k(x) = (x - e_k)^2 S(x). \tag{10}$$

In terms of this function the $2N$ branchpoints take the form

$$x_k^\pm = e_k \pm L\sqrt{S_k(e_k)} + ... \tag{11}$$

and we chose to let the corresponding N branchcuts to run between x_k^+ and x_k^-. We also chose the cycles A_k to surround x_k^+ and x_k^- on one sheet and the dual cycles B_k to run from x_k^+ to x_k^- on one sheet and back on the other sheet. Note that there may be corrections at this order but since we will only be interested in tree level prepotential we only use the zeroth order term $x_k^\pm = e_k$ although we depend on the existence of a branch cut.

Standard integration gives the first orders of the periods as

$$a_k = e_k + \frac{L^2}{4}\partial_k S_k + ... \tag{12}$$

which may also have corrections at this order. We are, however, only keeping the lowest order. The interesting terms of the dual periods turns out to be

$$2\sum_{i=1}^{N}(e_k-e_i)ln(e_k-e_i)-\sum_{i<j=1}^{N}(e_k+e_i+e_j)ln(e_k+e_i+e_j) \tag{13}$$

which should be compared to the derivative of the prepotential which is proportional to

$$2\sum_{i=1}^{N}(e_k-e_i)ln(e_k-e_i)-\sum_{i<j=1,i,j\neq k}^{N}(e_k+e_i+e_j)ln(e_k+e_i+e_j). \tag{14}$$

The difference is given by terms of the form $(2e_k+e_i)ln(2e_k+e_i)$ $i=1,...N,\ i\neq k$ which must be corrected by a counterterm of opposite sign. It seems obvious to change the residue function into

$$S(x)=\frac{\prod_{i<j=1}^{N}(x+e_i+e_j)}{\prod_{i=1}^{N}(2x+e_i)\prod_{i=1}^{N}(x-e_i)^2} \tag{15}$$

but this does not work for two reasons. It does not correspond to a hyperelliptic type of curve since the denominator is not a perfect square. Furthermore, as we will show below, there is no way to simultaneously get the correct R-parity and weight count using this form. Changing $(2x+e_i)$ into $(2x+e_i)^2$ gives a perfect square and may give correct R-parity but does not provide a correct weight count. There is a second possibility that satisfies the hyperelliptic curve demand namely

$$S(x)=\frac{\prod_{i<j=1}^{N}(x+e_i+e_j)}{\prod_{i=1}^{N}(x+e_i/2)^2\prod_{i=1}^{N}(x-e_i)^2}. \tag{16}$$

Both these forms contribute additional terms to the logarithmic parts of the prepotential by

$$\sum_{i=1}^{N}(2e_k+e_i)ln(2e_k+e_i) \tag{17}$$

but they have distinct R-parity contributions. Furthermore, there is an over count in (17) by $3e_kln e_k$ which is corrected for by multiplication by x^3. We hence suggest the form of the residue function

$$S(x)=\frac{x^3\prod_{i<j=1}^{N}(x+e_i+e_j)}{\prod_{i=1}^{N}(x-e_i)^2\prod_{i=1}^{N}(x+e_i/2)^2}. \tag{18}$$

The R-parity of $L^2S_k(x)$ should be 2 since we anticipate this form to contribute to the 1-instanton correction of the prepotential. Using the R-parity of L^2 which is $2N-(N-$

2)$(N-3)/2$ we find the form above (18) to have the correct R-parity. Note that the first suggestion (15) would give R-parity of $L^2 S_k(x)$ to be $2+N-3$. We could have introduced an additional x^{3-N} to ensure the correct R-parity but this would ruin the weight count.

We now have a new form of curve with functions

$$A(x) = \prod_{i=1}^{N}(x+e_i/2)\prod_{i=1}^{N}(x-e_i), \qquad B(x) = x^3 \prod_{i<j=1}^{N}(x+e_i+e_j). \tag{19}$$

This suggestion has, however, a serious flaw. As can be seen from the branchpoints discussion above this set of functions (19) forces us to consider more branchcuts centred on $x=-e_i/2$. This also means more cycles which will yield additional contributions to the prepotential. A closer study of the curve for the antisymmetric two tensor representation indicates a way out. If we multiply $A(x)$ by another factor of $\prod_{i=1}^{N}(x+e_i/2)$ and $B(x)$ by $\prod_{i=1}^{N}(x+e_i/2)^2$ then the ramification points where the two sheets coincide will not require branchcuts and hence there are no additional cycles. Furthermore, this does not change R-parity of the residue function $S(x)$.

The final form of the hyperelliptic approximation is thus

$$\begin{aligned} &y^2 + 2A(x)y + L^2B(x) = 0 && \text{where} \\ &A(x) = \prod_{i=1}^{N}(x+e_i/2)^2\prod_{i=1}^{N}(x-e_i) && B(x) = x^3\prod_{i=1}^{N}(x+e_i/2)^2\prod_{i<j=1}^{N}(x+e_i+e_j). \end{aligned} \tag{20}$$

This curve gives the correct 1-loop prepotential for $SU(N)$ with matter in the totally antisymmetric three tensor representation. It is also clear that by introducing a mass m for the matter representation in the usual way by taking $x \longrightarrow x+m$ and $e_i \longrightarrow e_i+m$ one may as usual integrate out m yielding the correct pure Yang-Mills result.

Defining matter may be incorporated by multiplication of additional factors

$$\prod_{i=1}^{N_f}(x+M_i) \tag{21}$$

to $B(x)$. Here $N_f \leq 5$ for $N=6$ and $N_f \leq 3$ for $N=7$ while $N=8$ does not allow for any defining matter.

We mentioned above the possibility to have another $B(x)$ with a wider range of indices e.g. $i \leq j = 1, ...N$ or $i, j = 1, ...N$ instead of $i < j = 1, ...N$ as above. We have not been able to find a form of $S(x)$ that would yield the correct number of weights and at the same time respect R-parity restrictions for any other choice of range of indices.

We now proceed by predicting the one instanton contribution to the prepotential. Following the same line of reasoning as in [4] we give that the following form of the one instanton contribution

$$F_{1-inst} = \sum_{k=1}^{N} (S_k(e_k) - 2\bar{S}_k(-e_k/2 - 3m/2)) \tag{22}$$

where

$$S(x) = \frac{(x+m)^3 \prod_{i<j=1}^{N}(x+e_i+e_j+3m) \prod_{j=1}^{N_f}(x+M_j)}{\prod_{i=1}^{N}(x+e_i/2+3m/2)^2 \prod_{i=1}^{N}(x-e_i)^2} \qquad \text{and} \tag{23}$$
$$S_k(x) \equiv (x-e_k)^2 S(x), \qquad \bar{S}_k(x) \equiv (x+e_k/2+3m/2)^2 S(x).$$

This result has the following properties. It yields the correct pure Yang-Mills and defining flavour results in the double scaling limit $m \to \infty$ and $L^2 m^{3+n(n-1)/2-2N} \to L_{new}^2$ where L_{new}^2 is the new scale of the theory. Furtermore, it does not have any poles as $e_k = -e_l/2 - 3m/2$ for some $l \in \{1, ...N\}$, which is the fact for a sum over $S_k(e_k)$ only. The precence of the sum over $\bar{S}_k(e_k)$ guarantees that there are only poles in the appropriate places.

It is clear that this curve is not the full curve. It is expected to have subleading (contributions with L^2 or higher order) terms in $A(x)$ as was the case for the antisymmetric rank two tensor. Also, the curve itself is expected to have a higher degree. We hope to be able to present more details on those issues in the near future.

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